Multiparty Homomorphic Encryption: from Theory to Practice

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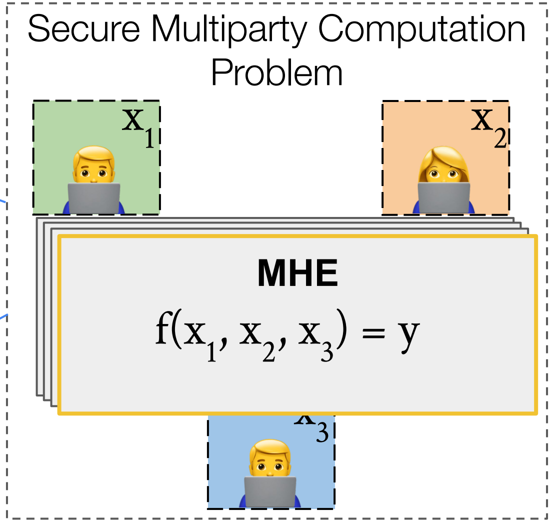
Secure Multiparty Computation

Multiple parties want to **compute** a public function **without disclosing** their inputs.



[Functionality]
"Output y"

[Input privacy]
"Without revealing more information about the inputs than what y does"

$\mathcal{P}$

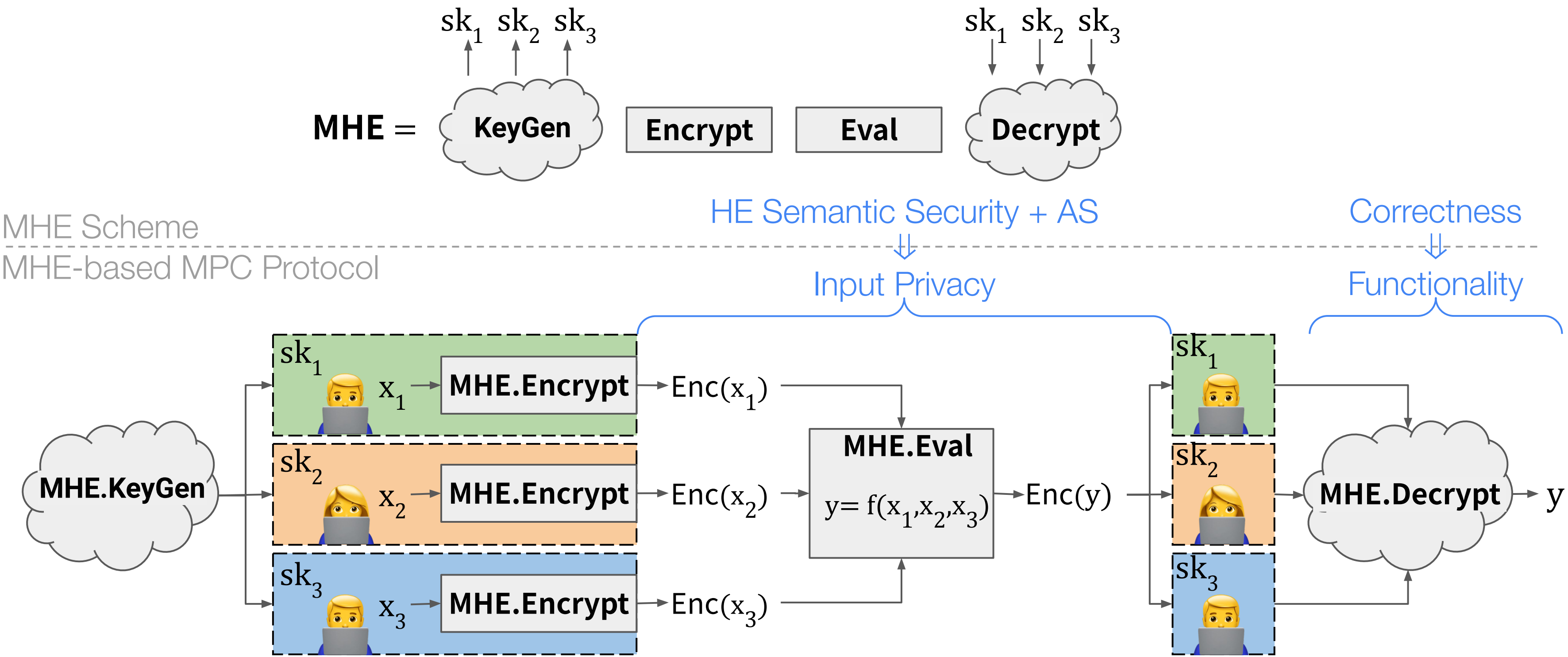
- N parties

$\mathscr{A} \subset \mathcal{P}$

- N-1 adversaries
- Passive and static

Multiparty Homomorphic Encryption – Intuition

Multiparty Homomorphic Encryption (MHE) extends Homomorphic Encryption (HE) with an **access-structure**.



sk_1 sk_2 sk_3 sk_1 sk_2 sk_3

MHE = **KeyGen** **Encrypt** **Eval** **Decrypt**

MHE Scheme

HE Semantic Security + AS

⇓

Correctness

⇓

MHE-based MPC Protocol

Input Privacy

Functionality

MHE.KeyGen

sk_1 x_1 → **MHE.Encrypt** → $Enc(x_1)$

sk_2 x_2 → **MHE.Encrypt** → $Enc(x_2)$

sk_3 x_3 → **MHE.Encrypt** → $Enc(x_3)$

MHE.Eval

$y= f(x_1,x_2,x_3)$

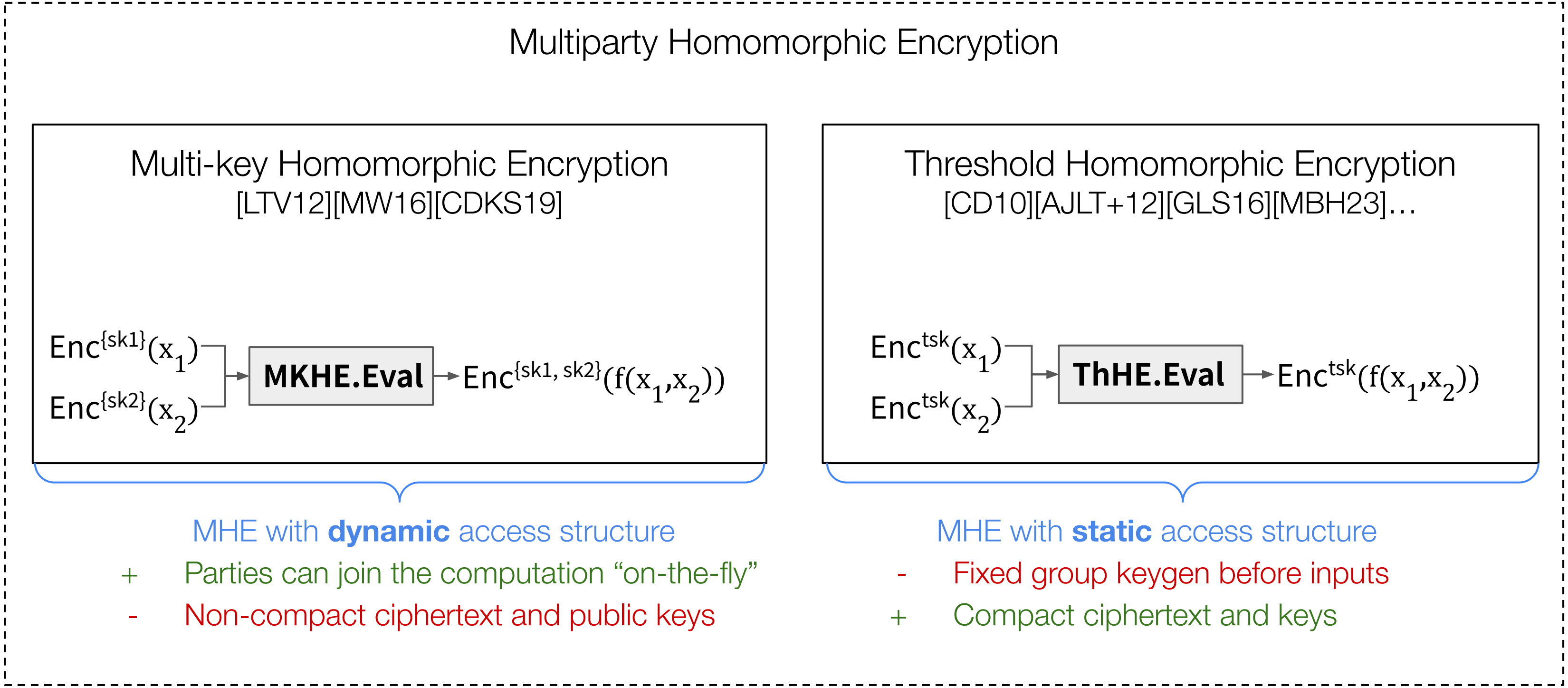
→ $Enc(y)$

sk_1 sk_2 sk_3

MHE.Decrypt → y

Multiparty Homomorphic Encryption – Two Main Families

There are two main families of MHE schemes.



Multiparty Homomorphic Encryption

Multi-key Homomorphic Encryption
[LTV12][MW16][CDKS19]

$\mathrm{Enc}^{\{sk1\}}(x_1)$, $\mathrm{Enc}^{\{sk2\}}(x_2)$ → **MKHE.Eval** → $\mathrm{Enc}^{\{sk1,\, sk2\}}(f(x_1,x_2))$

MHE with **dynamic** access structure

- \+ Parties can join the computation “on-the-fly”
- \- Non-compact ciphertext and public keys

Threshold Homomorphic Encryption
[CD10][AJLT+12][GLS16][MBH23]…

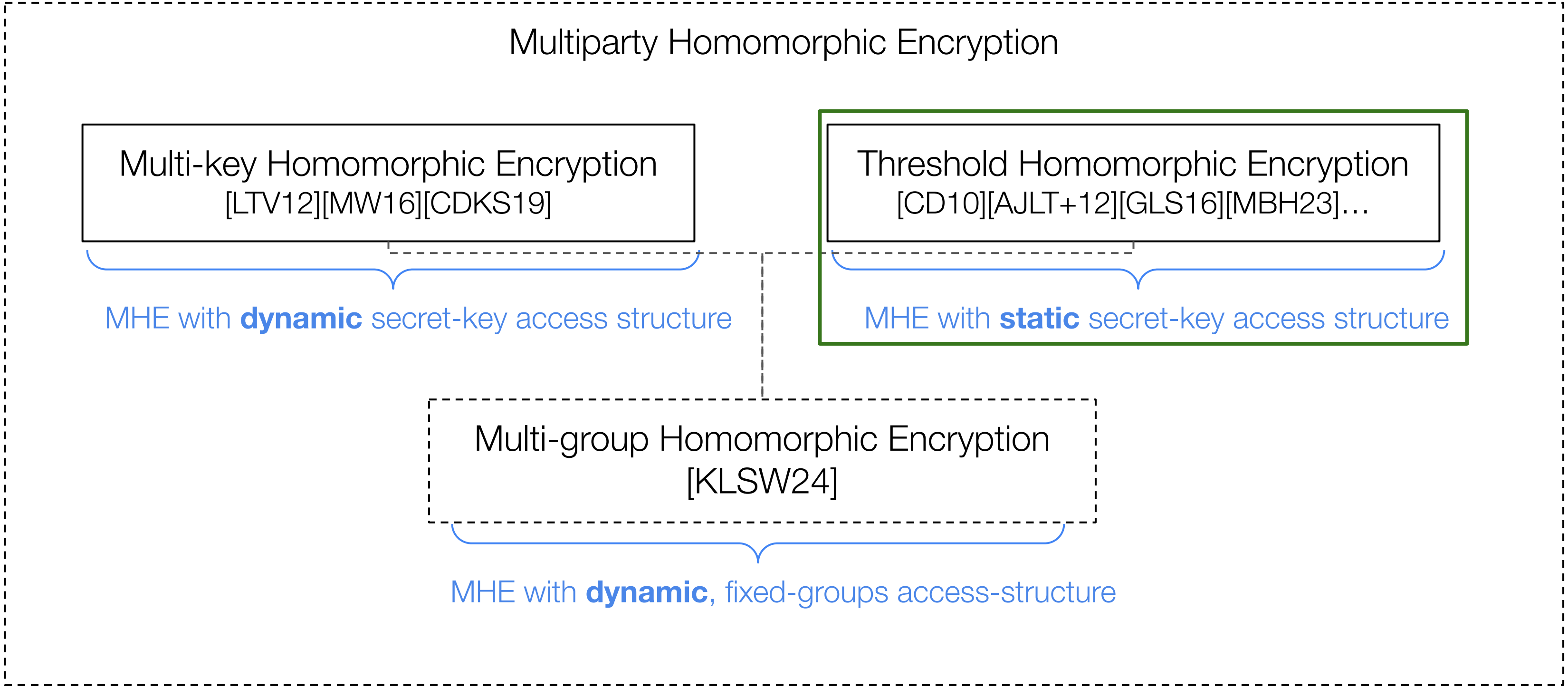
$\mathrm{Enc}^{tsk}(x_1)$, $\mathrm{Enc}^{tsk}(x_2)$ → **ThHE.Eval** → $\mathrm{Enc}^{tsk}(f(x_1,x_2))$

MHE with **static** access structure

- \- Fixed group keygen before inputs
- \+ Compact ciphertext and keys

Multiparty Homomorphic Encryption – Two Orthogonal Families

There are two main families of MHE schemes.



Multiparty Homomorphic Encryption

Multi-key Homomorphic Encryption
[LTV12][MW16][CDKS19]

MHE with **dynamic** secret-key access structure

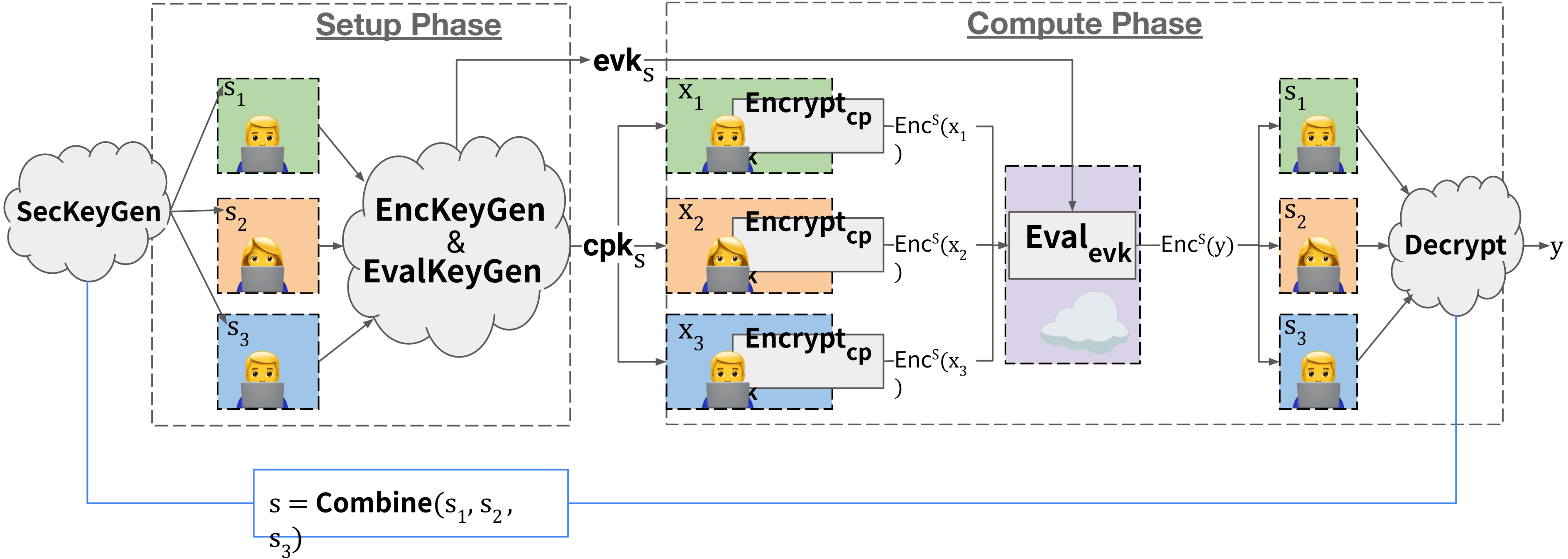
Threshold Homomorphic Encryption
[CD10][AJLT+12][GLS16][MBH23]…

MHE with **static** secret-key access structure

Multi-group Homomorphic Encryption
[KLSW24]

MHE with **dynamic**, fixed-groups access-structure

MHE-based MPC (Threshold-FHE case)



N-out-of-N-threshold: $s = s_1 + s_2 + s_3$

T-out-of-N-threshold: $s = \Delta_1 s_1 + \Delta_2 s_2$

Background: Ring learning-with-error [LPR10]

RLWE distribution:

Let:

$R = \mathbb{Z}_q[X]/(X^n+1)$	be a ring of degree n-1 polynomials with coefficients $\mathrm{mod}\ q$,
$\mathsf{U}(R)$	be the uniform distribution over R,
$\mathsf{Err}(R)$	be an error distribution over R $(\|\|e\|\| << q,\ e \leftarrow \mathsf{Err}(R))$,
$s \in R$	be a secret value in R

the *ring learning-with-error distribution* over s is defined as:

$$\mathsf{RLWE}_s := (sa + e,\ a) \qquad a \leftarrow \mathsf{U}(R) \qquad e \leftarrow \mathsf{Err}(R)$$

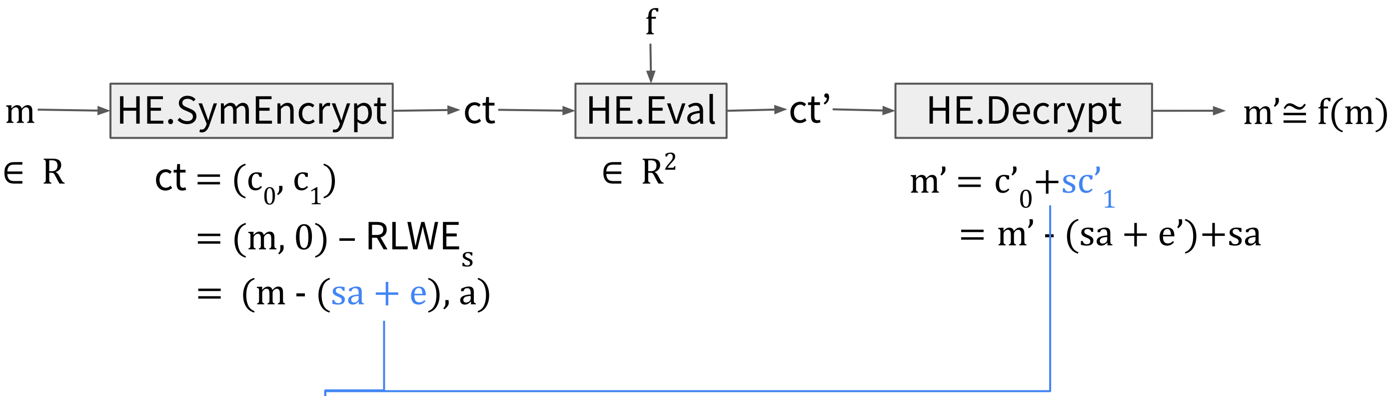
Given a polynomial number of <u>independent</u> samples from the RLWE_s distribution:

- **Search**: find s.
- **Decision**: distinguish from $\mathsf{U}(R^2)$

Background: (Symmetric) HE From RLWE

A simplified RLWE-based HE scheme.

Let $f: R \to R$, and $||s|| = 1$



$$\begin{aligned} ct &= (c_0, c_1) \\ &= (m, 0) - \mathrm{RLWE}_s \\ &= (m - (sa + e), a) \end{aligned}$$

$$\begin{aligned} m' &= c'_0 + sc'_1 \\ &= m' - (sa + e') + sa \end{aligned}$$

Scheme's operations are affine functions of the secret-key.

Secret-key operations are affine functions of the secret key

Other operations are also affine functions of the secret-key: $sa + e + x$

Setup phase:

Public Encryption Key Generation: $(sa + e, a)$

Public Rotation Key Generation for $\mathrm{rot}_k(\cdot)$: $(s\mathbf{b} + \mathbf{e} + \mathrm{rot}_{-k}(s)\mathbf{w}, \mathbf{b})$

Public Relinearization Key Generation: $(s\mathbf{d} + \mathbf{e} + s^2\mathbf{w}, \mathbf{d})$

Compute phase:

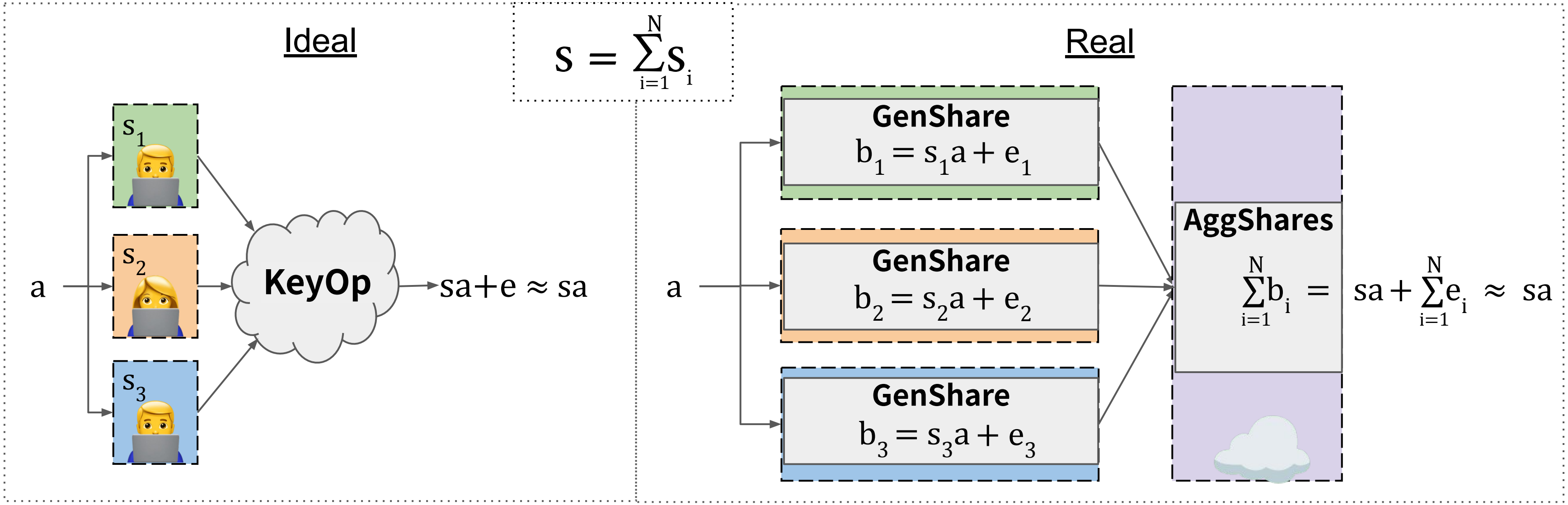
Decrypt: $sc_1 + e + c_0$

Re-encrypt: $((s\text{-}s')c_1 + e + c_0, c_1)$

MHE Scheme Construction – Secret-key Operations

Affine secret-key operations can be implemented as single-round protocols (Generalizing [AJLT+12][MTBH+21]).

→ We refer to these protocols as having **P**ublic **A**ggregatable **T**ranscripts (PAT)



Helper-Assisted, MHE-based MPC

The MHE-based MPC protocol has many practical advantages. [MTBH+21]

One-time setup

- ✓ Amortizable cost
- ✓ Session-like paradigm

Low communication complexity

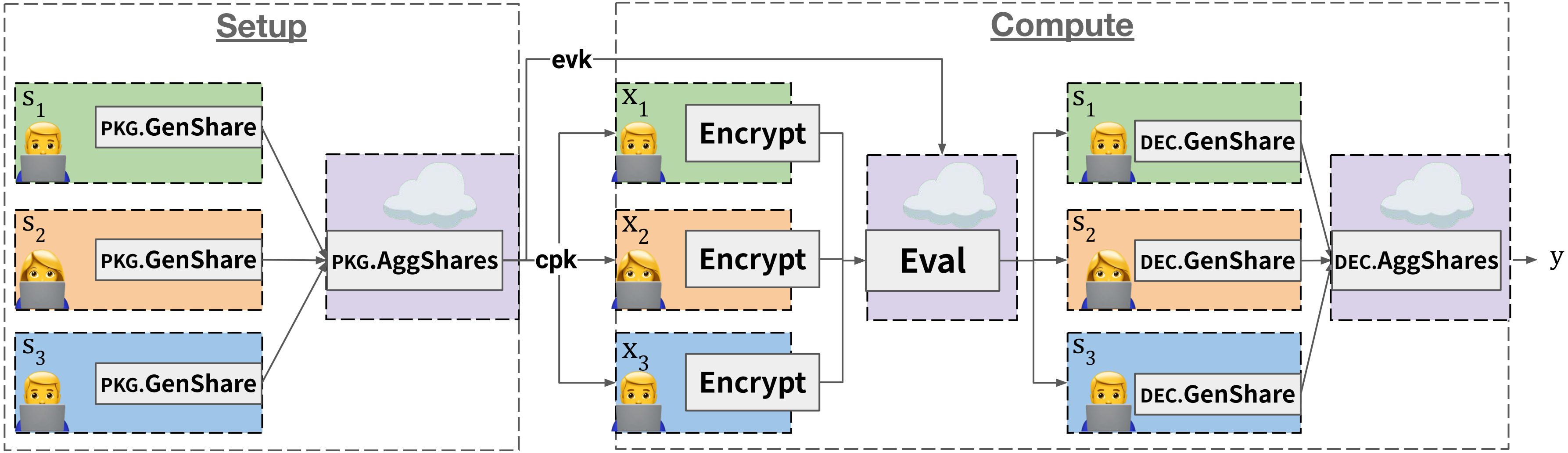
- ✓ 2+2 rounds
- ✓ Non-interactive Eval

Public Transcript

- ✓ Delegated public share aggregation
- ✓ Sublinear MPC

Delegated evaluation

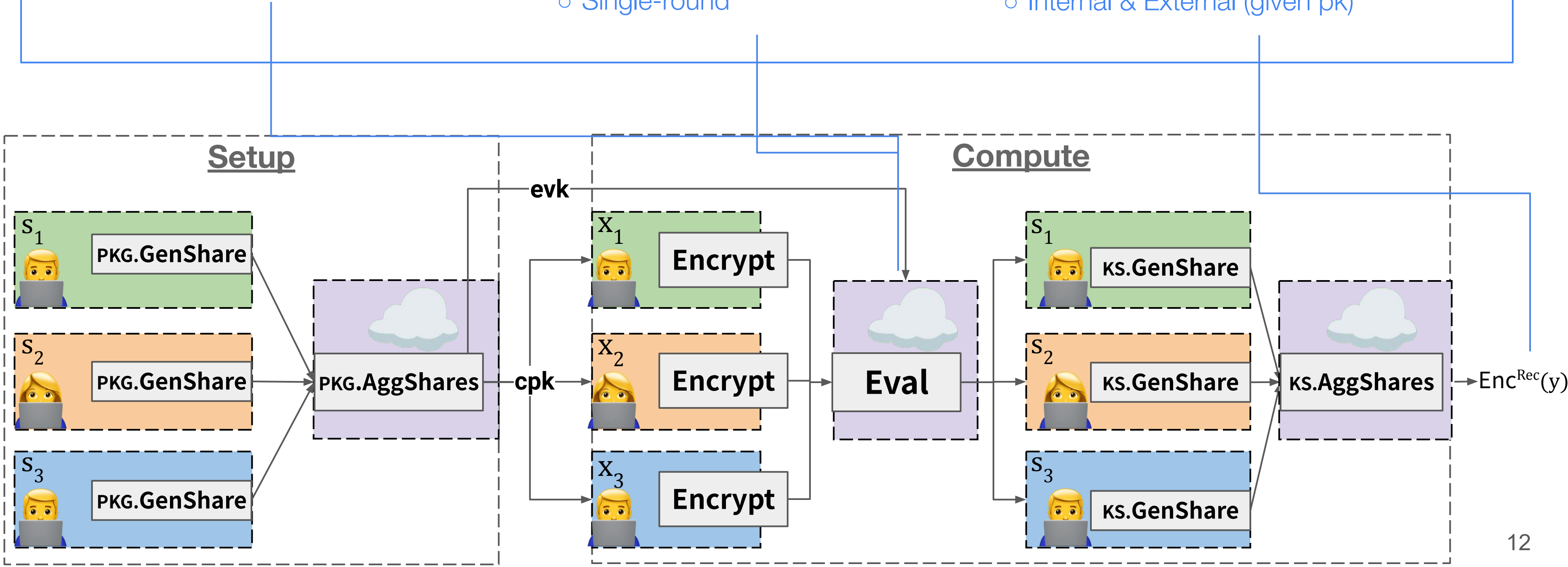
- ✓ In classic passive-adversary setting



Helper-Assisted, MHE-based MPC

Practicality Enhancements [**M**TBH21]

- ✓ Switching to/from data-level SS
  - ○ Single-round
- ✓ Refresh protocol
  - ○ Interactive “bootstrapping”
  - ○ Single-round
- ✓ Proxy-reencryption to design. receiver
  - ○ DEC → (P)KS
  - ○ Internal & External (given pk)



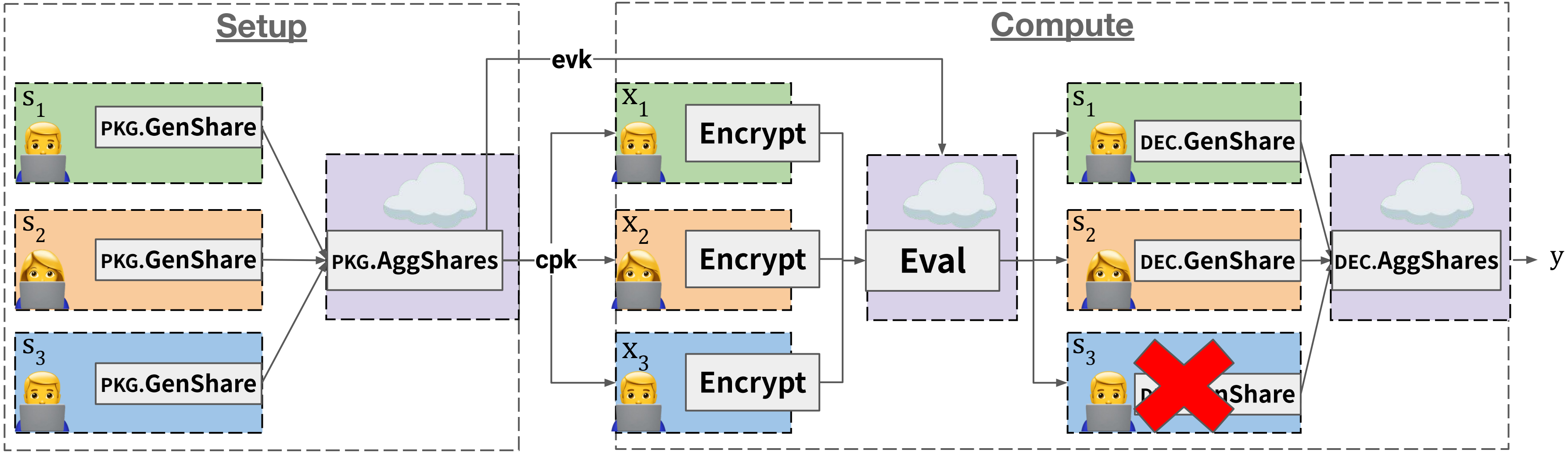
Helper-Assisted, MHE-based MPC

Fault-tolerance ?
- Temporary disconnects & Reboots
- Full crashes

"Not that bad"
- Most of the protocol state is public
- Delays the result

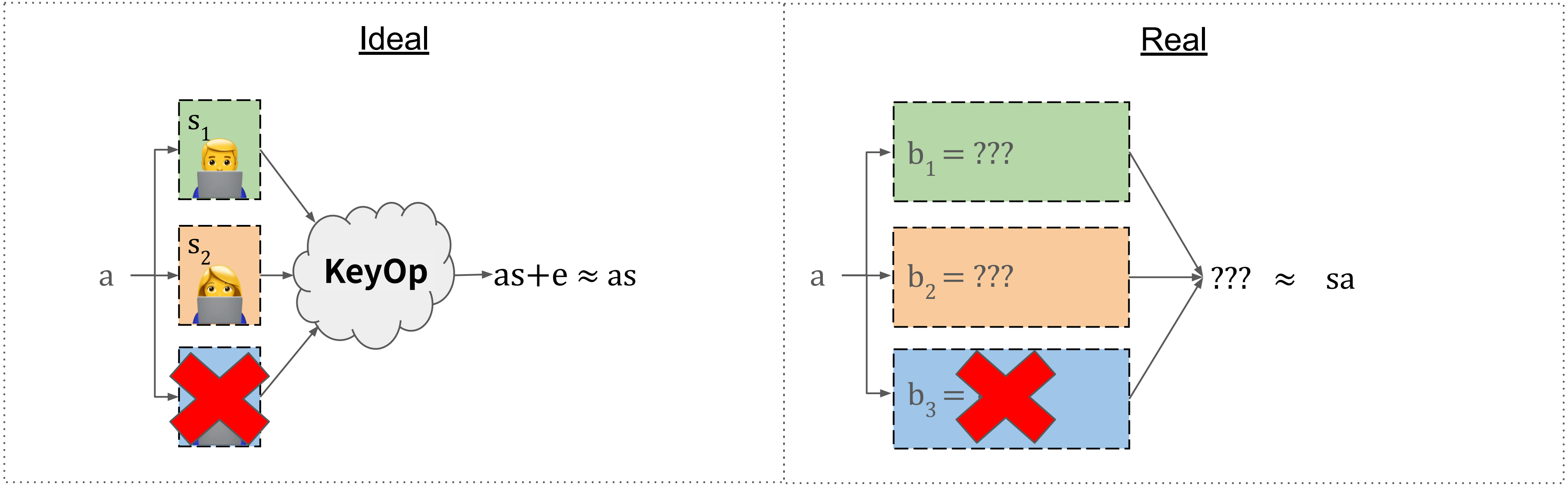
"Bad"
- … but "by design" when $|\mathcal{A}| = N\text{-}1$

→ What about $|\mathcal{A}| < T$?



T-out-of-N-Threshold Secret-Key Operations ?

Running PAT protocols among $\mathrm{T} < \mathrm{N}$ parties.



Previous Approaches

Previous approaches either require a trusted dealer or are leaky (and are all non-compact)

Compromises the secret-keys of offline parties
→ Compromises the “session”.

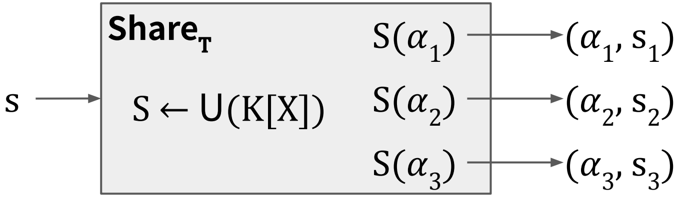
Requiring non-constant-size secrets
→ Leads to costly storage & ops.

Approach	**Trusted dealer**	**Leaky**	**Non-Compact**
1. Two-step [AJLT+12]	No	Yes	Yes
2. Single-step [BGGJ+18]	Yes	No	Yes
Ours [**M**BH23]	No	No	No

Shamir Secret-Sharing Scheme Reminder [Shamir 1979]

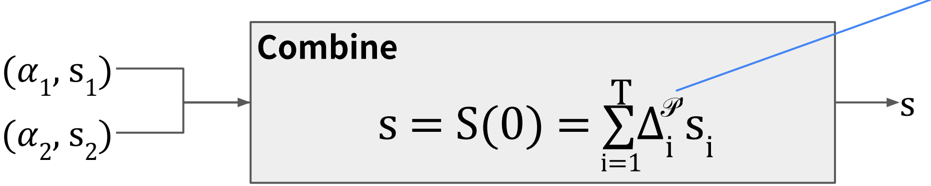
Shares are points on a uniformly random polynomial S over some finite field K where:

- S has degree-$(T\text{-}1)$ and
- $s = S(0)$.



The secret reconstruction is a linear combination of the shares with the **Lagrange interpolation coefficients:**

For $\mathcal{P}' \subset \mathcal{P}$ $|\mathcal{P}'| \geq T$ (w.l.o.g. assume $\mathcal{P}' = \{P_1, P_2, \ldots, P_T\}$):



$$s = S(0) = \sum_{i=1}^{T} \Delta_i^{\mathscr{P}} s_i$$

Lagrange coefficients depend on $\mathscr{P}$

$$\Delta_i^{\mathscr{P}} = \prod_{\substack{j=1 \\ j\neq i}}^{T} \alpha_j / (\alpha_j - \alpha_i)$$

Approach 1: Share Re-sharing + Two-steps Key-operations

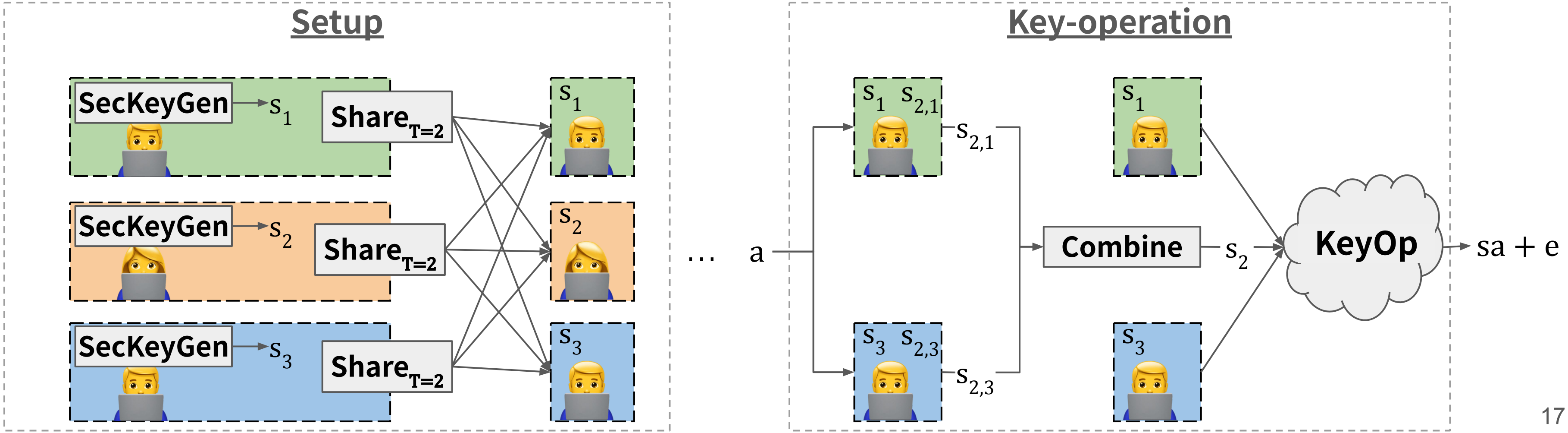
Asharov et al. proposed a share re-sharing scheme with two-steps key-operations [AJLT+12].

Pros

- ☑ No constraints on the field $\mathbb{K}$
- ☑ No need for trusted dealers

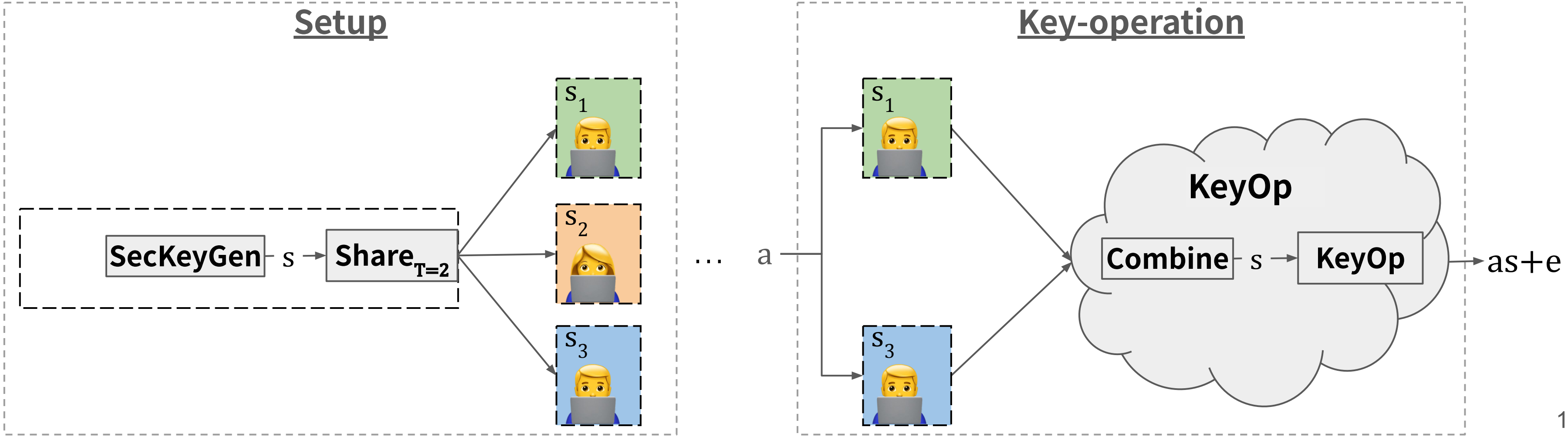
Cons

- ☒ Non-compact
- ☒ Leaks the failing parties' shares



Approach 2: Single-step Key-operation

Boneh et al. proposed a non-leaky approach based on a special sharing scheme ({0,1}-LSSS) [BGGJ+18].

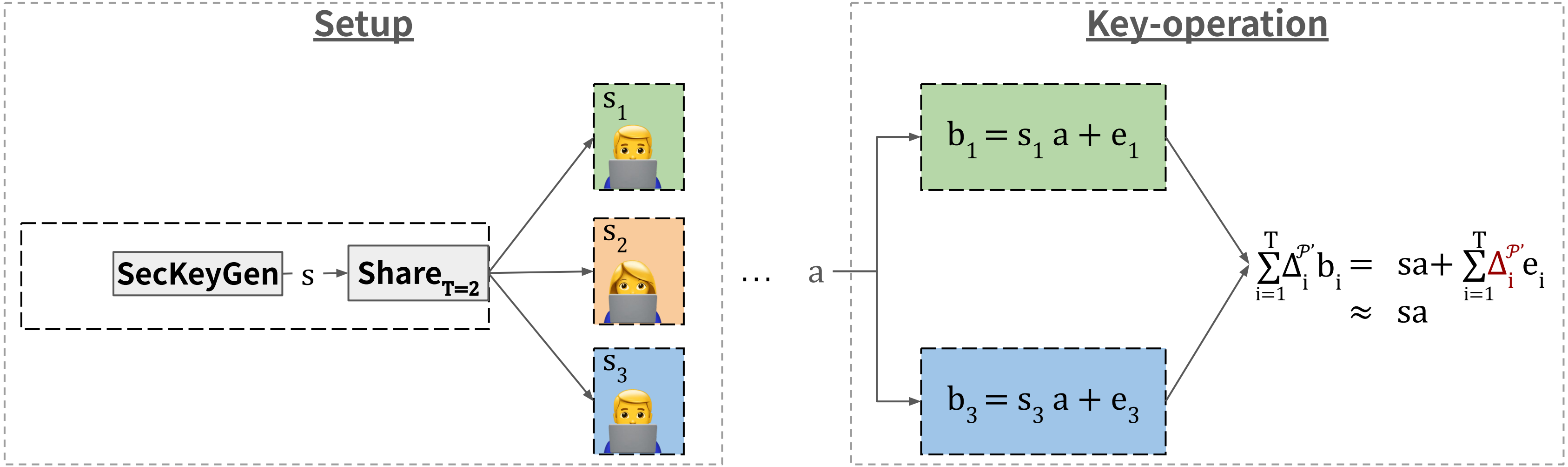


Approach 2: Single-step Key-operation

Boneh et al. proposed a non-leaky approach based on a special sharing scheme ({0,1}-LSSS) [BGGJ+18].

Pros
- ☑ Protects the failing parties' shares

Cons
- ☒ Non-compact ($O(N^{4.2})$)
- ☒ Requires a trusted dealer



Setup

SecKeyGen → s → **Share$_{T=2}$**

s_1

s_2

s_3

…

a

Key-operation

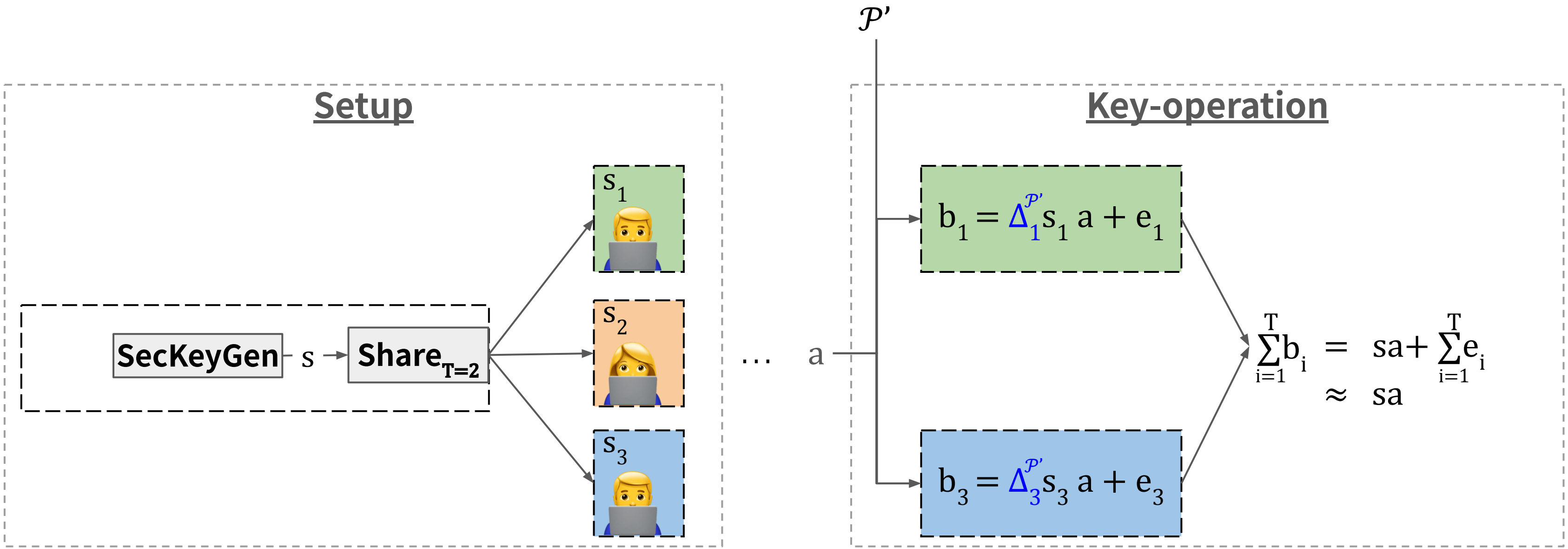
$b_1 = s_1 a + e_1$

$b_3 = s_3 a + e_3$

$$\sum_{i=1}^{T} \Delta_i^{\mathcal{P}'} b_i = sa + \sum_{i=1}^{T} \Delta_i^{\mathcal{P}'} e_i$$
$$\approx sa$$

Our Approach – Intuition

[MBH23]: If the parties know the set of online parties $\mathcal{P}'$ *before* computing their shares, there is a neat trick.



Setup

SecKeyGen s **Share$_{T=2}$**

s_1

s_2

s_3

…

a

$\mathcal{P}'$

Key-operation

$b_1 = \Delta_1^{\mathcal{P}'} s_1 a + e_1$

$b_3 = \Delta_3^{\mathcal{P}'} s_3 a + e_3$

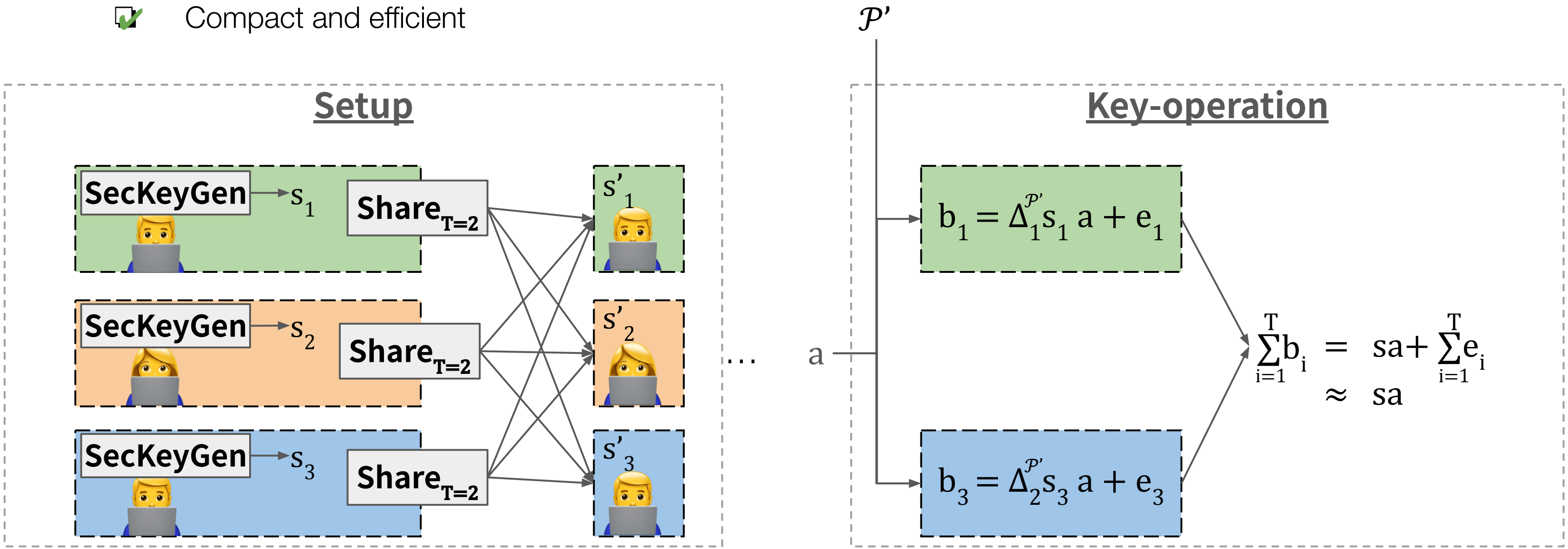
$$\sum_{i=1}^{T} b_i = sa + \sum_{i=1}^{T} e_i$$
$$\approx sa$$

Our Approach: Share Re-Sharing + Optimistic Key-operation

This trick combined with the share re-sharing approach yields an highly efficient solution. [MBH23]

Pros

- ☑ Protects the failing parties’ shares
- ☑ No trusted dealer
- ☑ Compact and efficient



Setup

SecKeyGen → s_1 **Share$_{T=2}$** → s'_1

SecKeyGen → s_2 **Share$_{T=2}$** → s'_2

SecKeyGen → s_3 **Share$_{T=2}$** → s'_3

…

Key-operation

$\mathcal{P}'$

a

$b_1 = \Delta_1^{\mathcal{P}'} s_1\ a + e_1$

$b_3 = \Delta_2^{\mathcal{P}'} s_3\ a + e_3$

$$\sum_{i=1}^{T} b_i = sa + \sum_{i=1}^{T} e_i \approx sa$$

Our Approach: Properties

Let:

$s = \sum^{N} s_i$ be the ideal secret-key in the N-out-of-N scheme

$s_{i,j} = S_i(\alpha_j)$ be the re-share of s_i held by party j in the T-out-of-N scheme

Then, for any $\mathcal{P}' \subset \mathcal{P}$, $|\mathcal{P}'| \geq T$, we can express S as:

$$s = \sum_{i=1}^{N} s_i = \sum_{i=1}^{N}\sum_{j=1}^{T}\Delta_j^{\mathcal{P}'} s_{i,j} = \sum_{j=1}^{T}\boxed{\sum_{i=1}^{N}\Delta_j^{\mathcal{P}'} s_{i,j}} = \sum_{j=1}^{T}\Delta_j^{\mathcal{P}'}\boxed{\sum_{i=1}^{N} s_{i,j}} = \boxed{\sum_{j=1}^{T} s_j^{\mathcal{P}'}}$$

Sum-over-T terms contains re-shares held by party j only.

Sum-over-N terms do not depend on the set of online parties $\mathcal{P}'$.

T-out-of-T additive sharing of the collective secret-key.

Non-leakyness / Correctness

The each party can compute its share locally given $\mathcal{P}'$.

Compactness

T-out-of-N re-shares can be aggregated in the setup phase.

Modularity

We can use the protocols of the N-out-of-N scheme for $N=T$.

Our Approach: Discussion

The dependence on the online parties' oracle introduces **two requirements**:

1. Implementation of the oracle

- Good $\mathcal{P}'$ requires accurate view over the network
- Requires consensus on the participant set $\mathcal{P}'$

2. A protocol-failure handling mechanism

- Parties can fail *after* $\mathcal{P}'$ was issued.
- Requires defining a (synchronous) "protocol failure" event.

Approach	**Trusted dealer**	**Leaky**	**Compact**	**Asynchronous**
Two-step [AJLT+12]	No	Yes	No	No
Single-step [BGGJ+18]	Yes	No	No	yes
Ours [**M**BH23]	No	No	Yes	No

Upside: Req. 1. + 2. can be realized in the passive, synchronous setting. So our scheme is highly relevant in this setting.

Downside: our method does not "directly" apply to stronger settings.

Implementation

Both the N-out-of-N- and the T-out-of-N-threshold scheme are implemented in Lattigo [**M**BTH20]

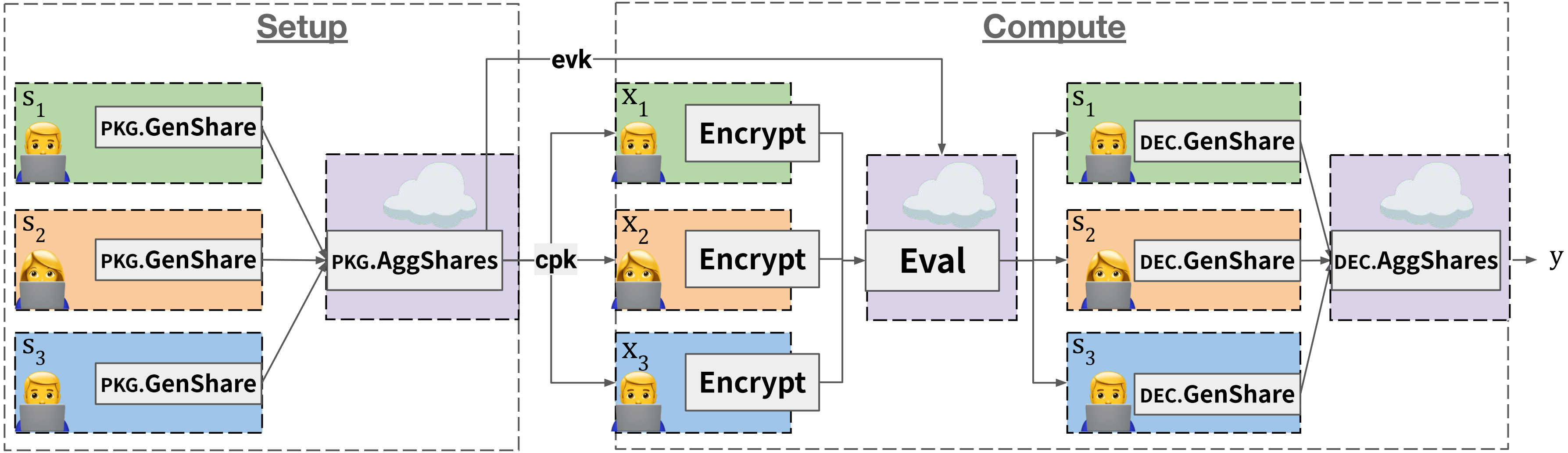
- Supports: (Threshold-) BGV, BFV, CKKS
- (Single-party) CKKS bootstrapping
- Standalone, modular math & crypto layers
- **Open-source** (Apache 2.0 licence)
- 100% written in Go
- Full suite of unit-test and benchmarks

LATTIGO

https://github.com/tuneinsight/lattigo

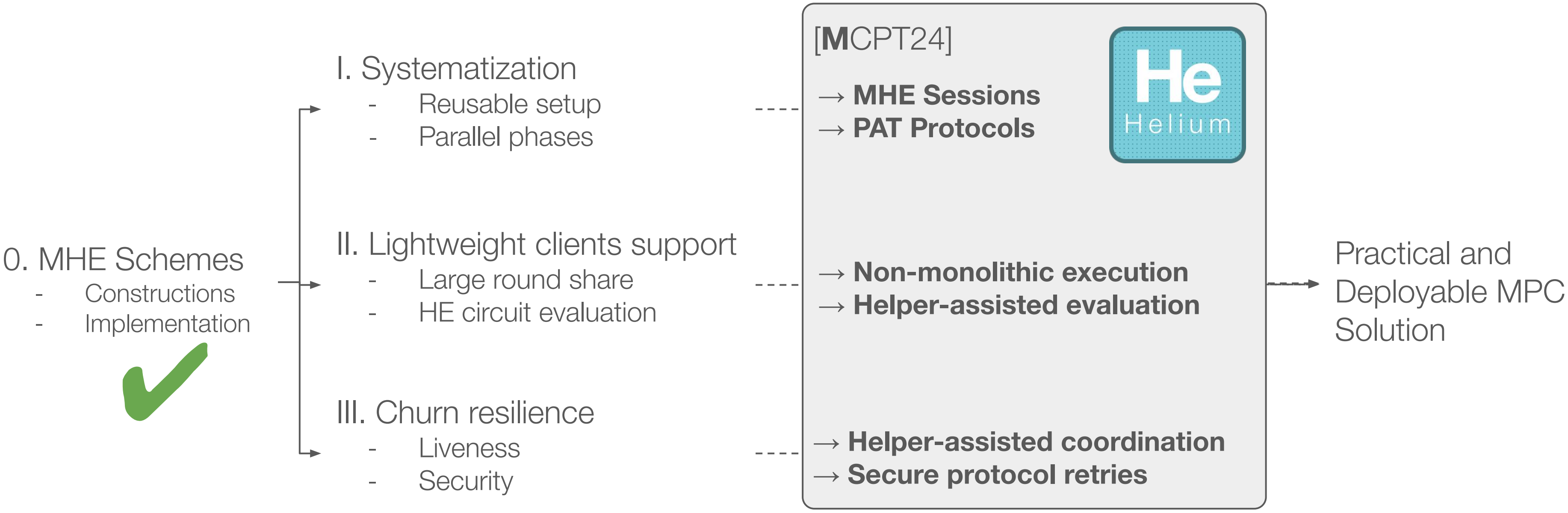
Fault-tolerant MHE-based MPC Protocol

Lattigo & OpenFHE provide the core element of the MHE-based MPC protocol…



Practical Challenges of MHE-based MPC

…, but the way to practice is full of challenges.



0. MHE Schemes
- Constructions
- Implementation

✓

I. Systematization
- Reusable setup
- Parallel phases

II. Lightweight clients support
- Large round share
- HE circuit evaluation

III. Churn resilience
- Liveness
- Security

[**M**CPT24]

He
Helium

→ **MHE Sessions**
→ **PAT Protocols**

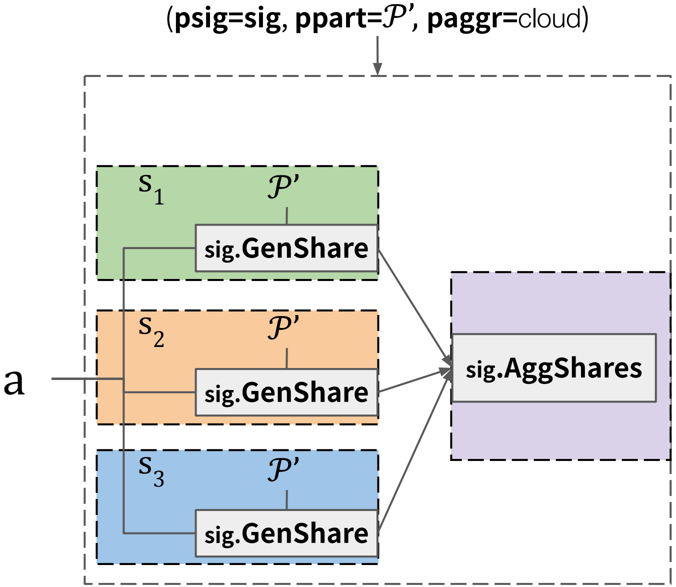
→ **Non-monolithic execution**
→ **Helper-assisted evaluation**

→ **Helper-assisted coordination**
→ **Secure protocol retries**

Practical and Deployable MPC Solution

Helium: Systematization

MHE-MPC reduces to running many **P**ublic **A**ggregatable **T**ranscript (PAT) **protocols** within a **session.**



PAT-protocol "mini-language":

psig := (**ptype**, **pargs**)

ptype ∈ {**CKG**, **EKG**, **DEC**, **KS**}

pargs: free protocols arguments
- operation identifier for **EKG**
- ciphertext identifier for **DEC/KS**

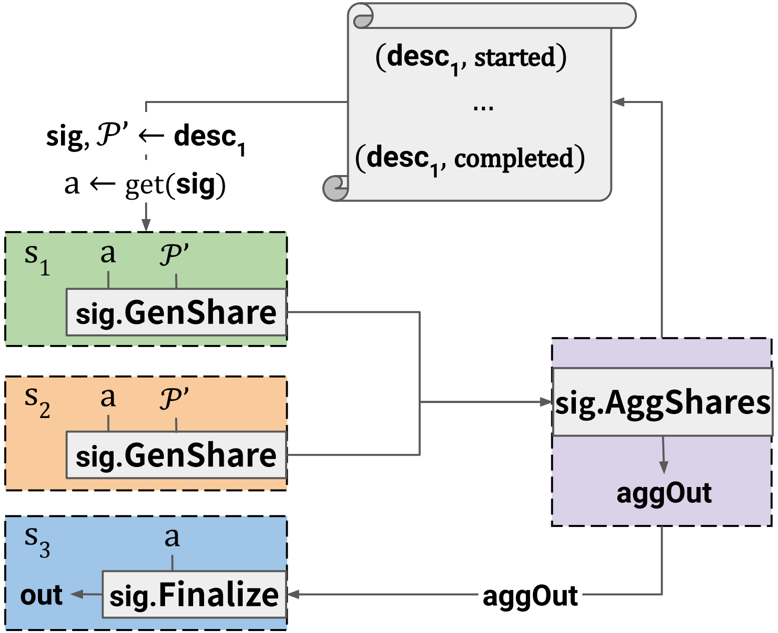
pdesc := (**psig**, **ppart, paggr**)

ppart $= \mathcal{P}' \subset \mathcal{P}, |\mathcal{P}'| = \mathrm{T}$

paggr: aggregator's network identity

Helium: Helper coordination

Helper orchestrates the execution via a **compact** public coordination log.

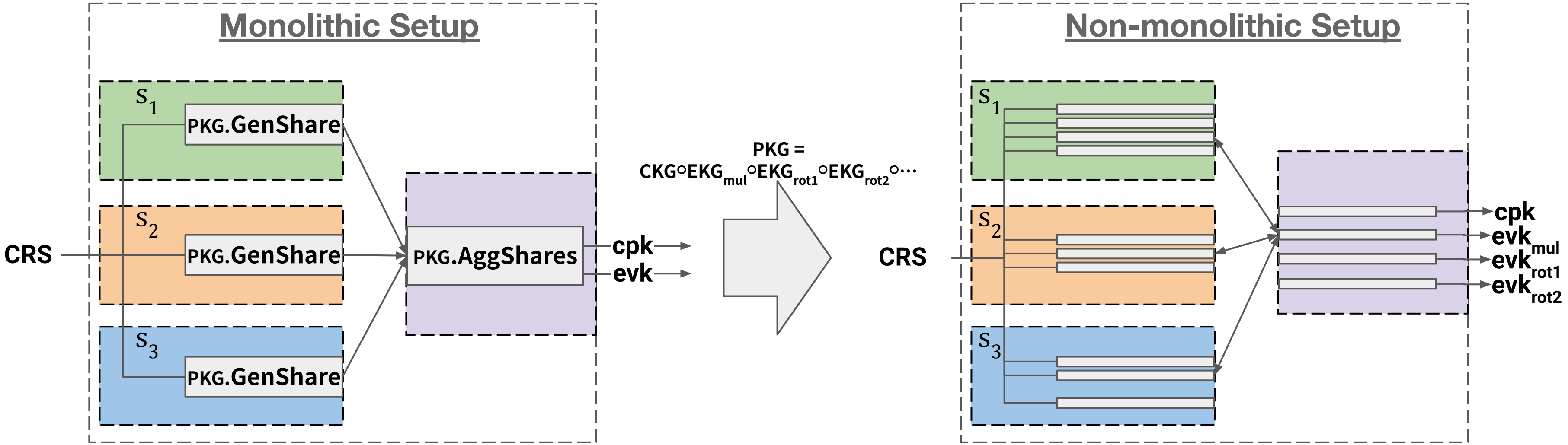


coordmsg := (**pdesc**, **pstatus**)

pstatus ∈ {**started**, **completed**}

coordlog := coordmsg || coordlog

Helium: Non-monolithic execution

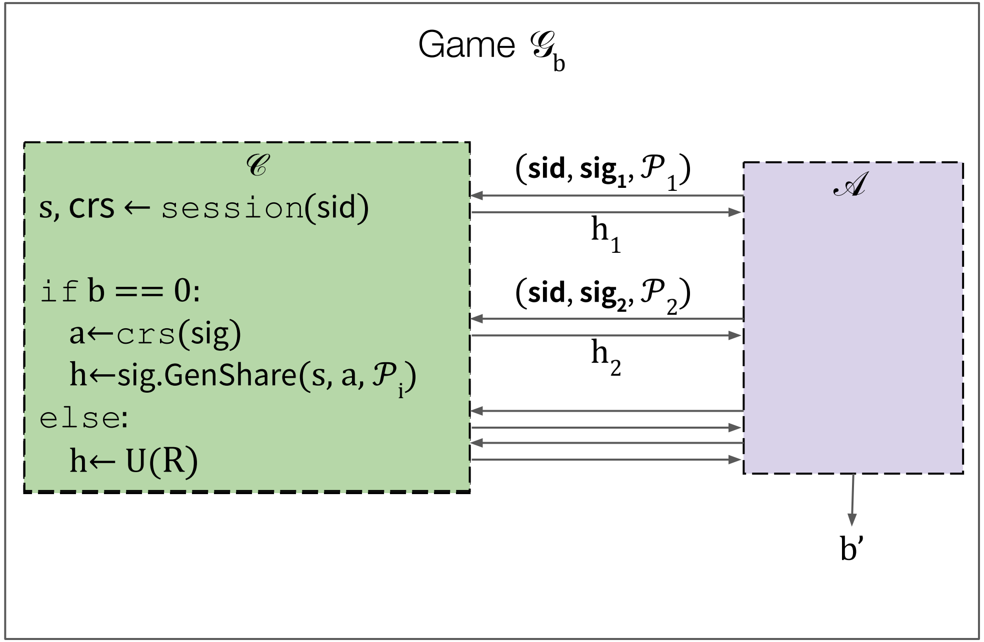


- Potentially **large round-1 message**
- Round can **fail** in the T-out-of-N Setting
- **Cannot distribute** the load if N_{online} > T
- No need for coordination: 1-2 round
- Matches the theoretical formulation

- **Small messages**: support weak nodes
- Dynamic Participant sets: **adapt** to network
- **Enables distribution** of the load
- Requires (finer grained) coordination
- Execution no longer match model of security proof

Helium: Modelling the Non-Monolithic Execution

Modelling the protocol execution mechanism as an interactive game (keygen case).



```
s, crs ← session(sid)

if b == 0:
  a←crs(sig)
  h←sig.GenShare(s, a, $\mathcal{P}_i$)
else:
  h← U(R)
```

- Security from RLWE assumption

$$\text{sig.GenShare}(s, a, \mathcal{P}) \sim \Delta^{\mathcal{P}} sa + e$$

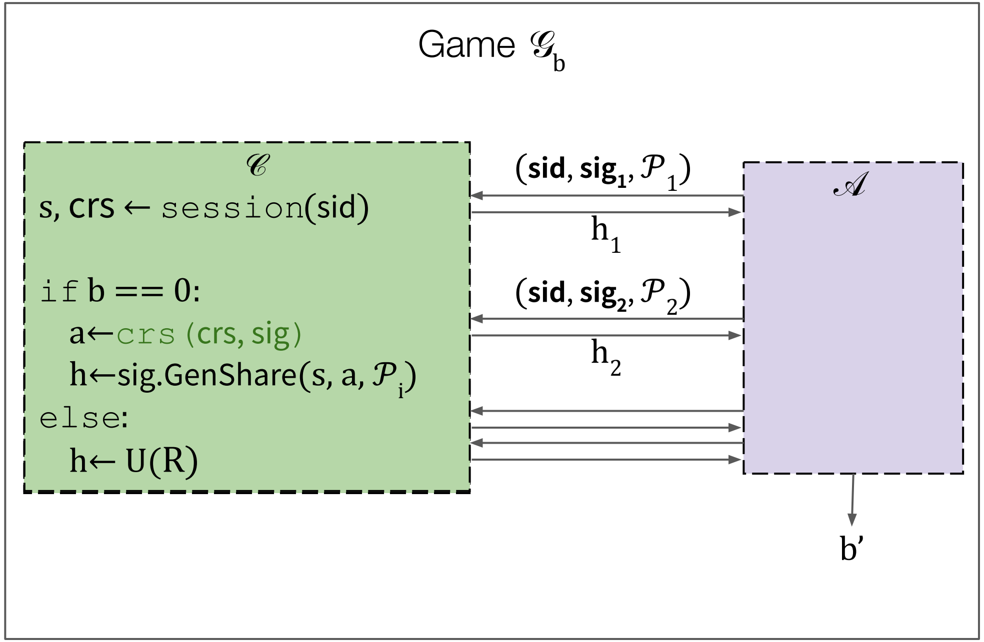
→ holds if $\mathscr{A}$ can only query a <u>poly. number</u> of <u>indep.</u> samples

→ a, e must be fresh

→ a is read from the CRS

Helium: Modelling the Non-Monolithic Execution

A non-monolithic, adaptive execution requires a random-access CRS



Game $\mathscr{G}_b$

$\mathscr{C}$

```
s, crs ← session(sid)

if b == 0:
    a ← crs(crs, sig)
    h ← sig.GenShare(s, a, $\mathcal{P}_i$)
else:
    h ← U(R)
```

$(\mathbf{sid}, \mathbf{sig_1}, \mathcal{P}_1)$

h_1

$(\mathbf{sid}, \mathbf{sig_2}, \mathcal{P}_2)$

h_2

$\mathscr{A}$

b’

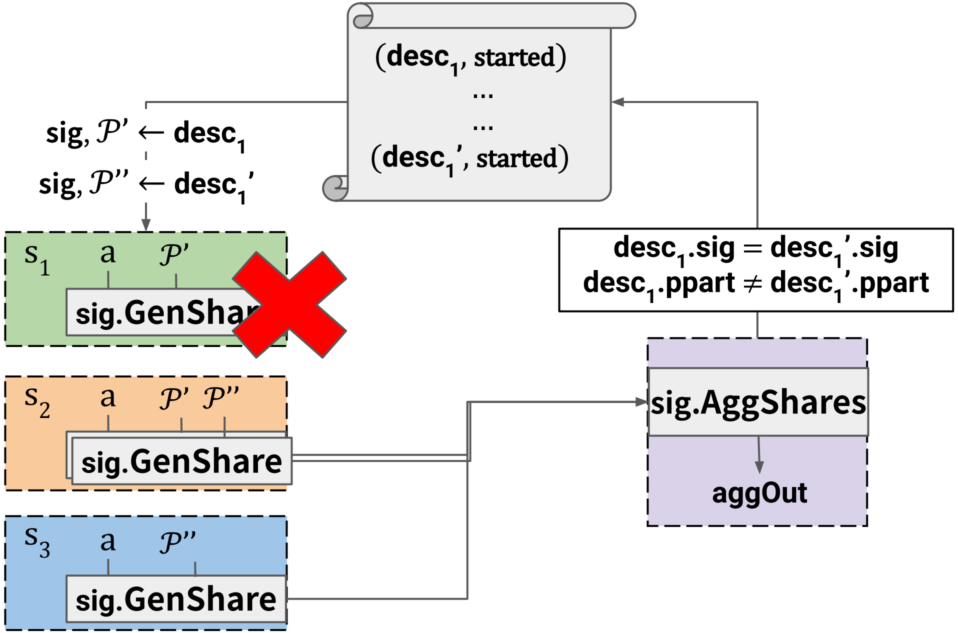
- a must be read fresh from the CRS for each sig.
- Not all parties participate to all protocols (or are even online) → Need a random-access CRS
- “Branching” the base CRS for each signature:

$$\texttt{crs}(\text{crs}, \text{sig}) \;:=\; \texttt{xof}(\text{crs}||\text{sig})$$

Unique signatures → fresh public polynomials

Helium: Helper coordination – Retries

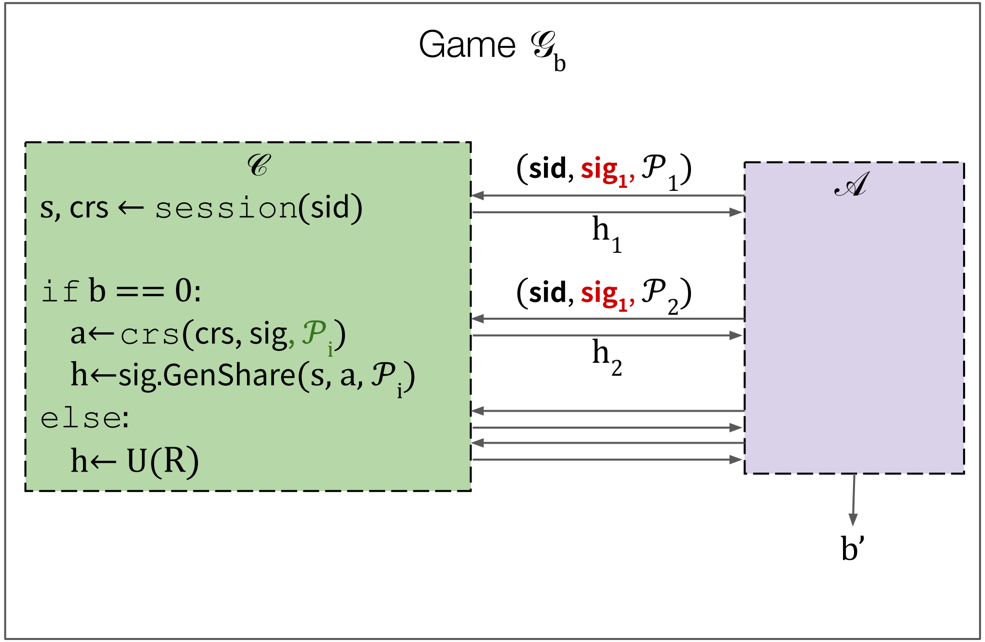
The PAT protocol semantic and non-monolithic execution provides a natural retry mechanism.



- \+ Minimal extra logic for retries
- \- Protocol failures require providing the challenger with more freedom.

Helium: Modelling the Non-Monolithic Execution

Retries allow repeated signatures with different participant sets.



Game $\mathscr{G}_b$

$\mathscr{C}$

```
s, crs ← session(sid)

if b == 0:
    a ← crs(crs, sig, $\mathcal{P}_i$)
    h ← sig.GenShare(s, a, $\mathcal{P}_i$)
else:
    h ← U(R)
```

- CRS-sampled polynomials are no longer fresh

$$(h_1, h_2, a)=(\Delta^{\mathcal{P}_1} sa + e_1 , \Delta^{\mathcal{P}_2} sa + e_2, a) \overset{c}{\not\equiv} U(R^3)$$

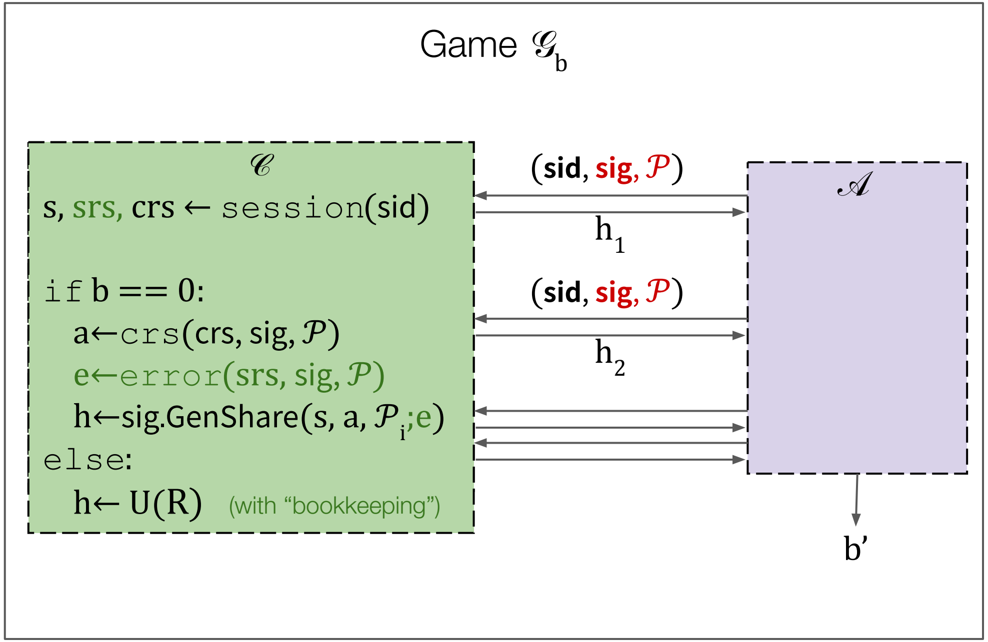
- “Branching” the base CRS for each protocols:

$$\texttt{crs}(\text{crs}, \text{sig}, \mathcal{P}) := \texttt{xof}(\text{crs} || \text{sig} || H(\mathcal{P}))$$

Unique protocol descriptor → fresh public polynomials

Helium: Modelling the Non-Monolithic Execution

Retries allow repeated signatures with <u>same participant sets</u>:



Game $\mathscr{G}_b$

$\mathscr{C}$

```
$s$, $srs$, $crs \leftarrow$ session(sid)

if $b == 0$:
    $a \leftarrow$ crs(crs, sig, $\mathcal{P}$)
    $e \leftarrow$ error(srs, sig, $\mathcal{P}$)
    $h \leftarrow$ sig.GenShare(s, a, $\mathcal{P}_i$;e)
else:
    $h \leftarrow U(R)$   (with "bookkeeping")
```

- Can happen in passive adv. setting:
    1. The network state at retry time.
    2. Stateless node restart.

$$(h_1, h_2, a) = (\Delta^{\mathcal{P}} sa + e_1 , \Delta^{\mathcal{P}} sa + e_2, a) \overset{c}{\not\approx} U(R^3)$$

- Bad solution: retry sequence numbers
  → Does not prevent case 2 failure.

- **Better solution**: resettable PAT protocols
  → By seeding the error distribution

→ Ensure $\mathscr{C}$ behaves like a random function

Practical Challenges of MHE-based MPC

0. MHE Schemes ✓
  - Construction
  - Implementation

I. Systematization ✓
  - Sessions
  - PAT protocol abstraction

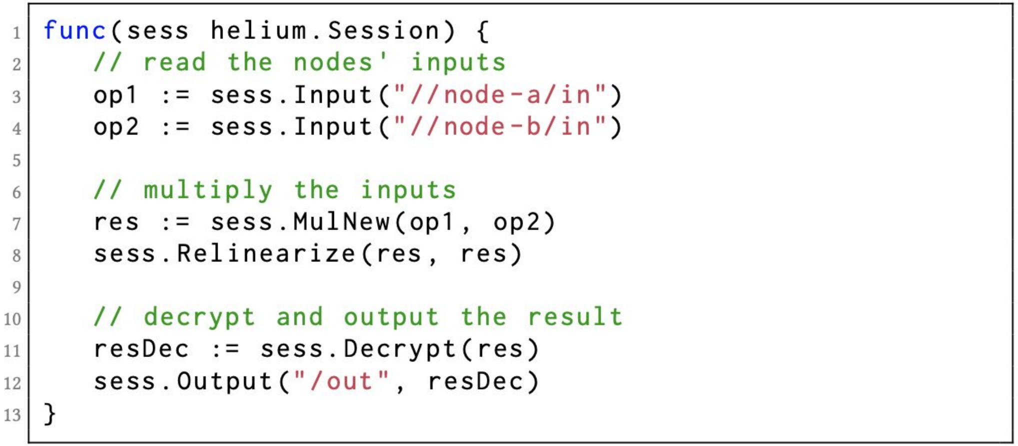
II. Lightweight clients support ✓
  - Non-Monolithic Execution
  - O(1) participant overhead

III. Churn resilience ✓
  - Retries
  - Resettability

Practical and Deployable MPC Solution

Implementation

We implemented Helium as an open-source library.

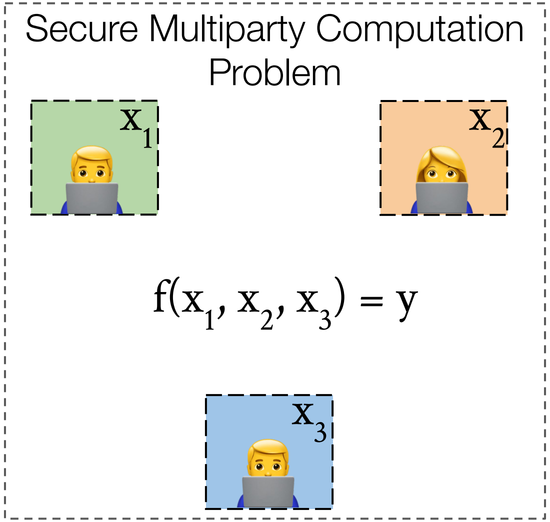


```
func(sess helium.Session) {
    // read the nodes' inputs
    op1 := sess.Input("//node-a/in")
    op2 := sess.Input("//node-b/in")

    // multiply the inputs
    res := sess.MulNew(op1, op2)
    sess.Relinearize(res, res)

    // decrypt and output the result
    resDec := sess.Decrypt(res)
    sess.Output("/out", resDec)
}
```

https://github.com/ChristianMct/helium

Conclusion: My “FHE:IDEA”

Hot take: In the short/medium term, future deployments of FHE will be solving SMPC problems.



Secure Multiparty Computation Problem

$$f(x_1, x_2, x_3) = y$$

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